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Linear algebra

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Problem Set # 10

Exercise 1($\star$):
Prove that for A, $B \in M_n(K)$, we have:

1. $||\lambda A||_\infty = |\lambda| \cdot ||A||_\infty$, all $\lambda \in K$,
2. $||A + B||_\infty \leq ||A||_\infty + ||B||_\infty$;
3. $||AB||_\infty \leq n \cdot ||A||_\infty ||B||_\infty$.

Exercise 2($\star$):
All limits are seen in $|| \cdot ||_\infty$-norm as $n \to \infty$. If $A_n \to A$ and $B_n \to B$ and $\lambda_n \to \lambda$ in $\mathbb{C}$. Prove that:

1. $A_n + B_n \to A + B$
2. $A_n B \to AB$ and $AB_n \to AB$;
3. $A_n B_n \to AB$ (matrix multiplication is jointly continuous operator in its two imputs).
   Hint: Add and subtract $A_n B$, then apply the triangle inequality.
4. $QA_nQ^{-1} \to QAQ^{-1}$ for A square matrix and Q invertible matrix.
5. $\lambda_n A_n \to \lambda A$.

Exercise 3($\star$):
Determine whether the matrix

$$A = \begin{pmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ 6 & -6 & 3 \end{pmatrix}$$

is diagonalizable over $K = \mathbb{R}$ or $\mathbb{C}$. In any case, find basis vectors for eigenspace $E_\lambda(L_A)$.

Exercise 4($\star$):
The matrix

$$A = \begin{pmatrix} 1 & 0 & -1 & 3 \\ 2 & -1 & 0 & 7 \\ 1 & -2 & 3 & 5 \end{pmatrix}$$

is row equivalent to

$$\begin{pmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Its transpose is row equivalent to

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now consider the linear map $L_A : V \to W$ between the vector spaces $V = K^4$ and $W = K^3$.

1. Find bases for $K(L_A) \subseteq V$ and $R(L_A) \subseteq W$.
2. Find bases $\mathcal{X}$, $\mathcal{N}$ for V, W such that $[L_A]_{\mathcal{N},\mathcal{X}}$ takes the form

$$\left(\begin{array}{ccc|c} 1 & & 0 & \\ & \ddots & & 0 \\ 0 & & 1 & \\ \hline & 0 & & 0 \end{array}\right)$$

Exercise 5($\star$):
If the linear operator $T : \mathbb{R}^3 \to \mathbb{R}^2$ has matrix

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 0 & -1 \end{pmatrix}$$

with repect to the standard basis, find the matrix $[T]_{\mathcal{N},\mathcal{U}}$ of the operation with respect to the new bases

$$\mathcal{U} : u_1 = (0, 1, -1),\ u_2 = (2, 2, -1),\ u_3 = (4, 0, 1)$$

$$\mathcal{N} : v_1 = (1, -1),\ v_2 = (0, -1)$$